

# Pseudorandom Signatures

## Relations among Privacy Notions for Digital Signatures



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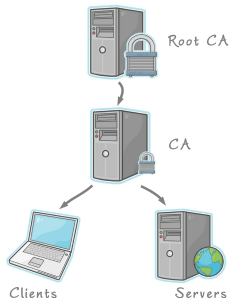
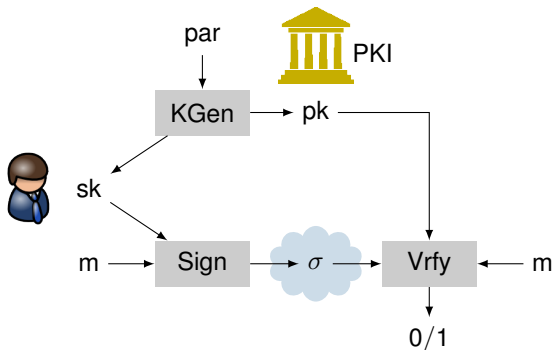
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joint work with

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Saarland University (Germany), University of Surrey (UK), Royal Holloway University of London (UK)

# Digital Signatures

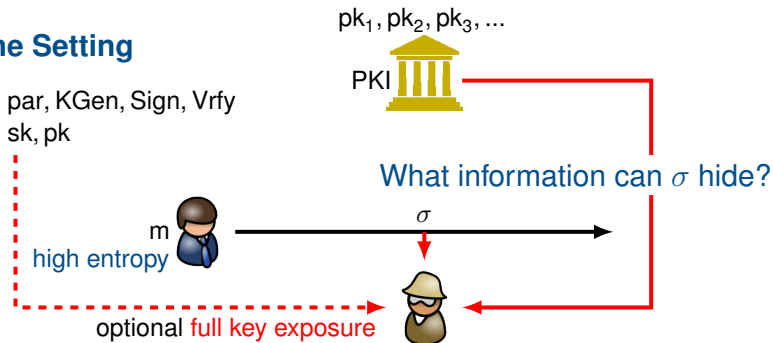


Digital signatures **do not offer privacy!**  
... due to public verification.

Is privacy for digital signatures thus hopeless?

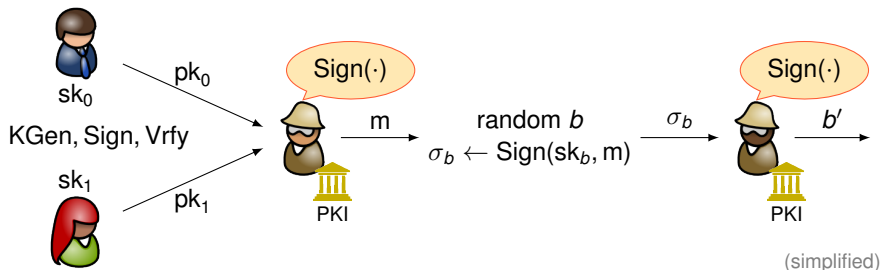
Not quite!

## The Setting



## Anonymous Signatures (ANON)

- ▶ Yang, Wong, Deng, Wang @ PKC 2006
- ▶ Fischlin @ PKC 2007
- ▶ Bellare, Duan @ eprint 2009 (non-standard signatures)
- ▶ Saraswat, Yun @ ProvSec 2009 (non-standard signatures)
- ▶ Zhang, Imai @ IEICE Trans. 92-A 2009 (non-standard signatures)





# Applications and Theoretical Aspects of Privacy-Friendly Signatures

Signatures are often sent together with signed message.

However, **anonymous/confidential signatures** are useful in

- ▶ **anonymous auctions** (where bid is revealed later)
- ▶ **anonymous key exchange**
- ▶ **output signing of secure multi-party computation**



But signatures (e.g., in European passports [Bringer et al. @ ACNS 2010 ]) might already be **distinguishable by the signing algorithm and parameters** used

Existence of **anonymous/confidential signatures** also raises theoretical questions:

- ▶ How are ANON and CONF **related**?
- ▶ Can signature schemes achieve **both** ANON and CONF?
- ▶ Is there a **limit** on the information that can be hidden?



# Relationship between ANON and CONF

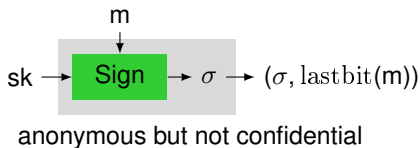
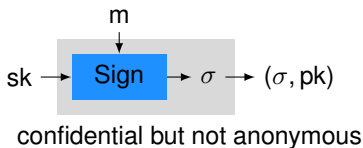
We have **two privacy notions**, but the work isn't complete...

ANON

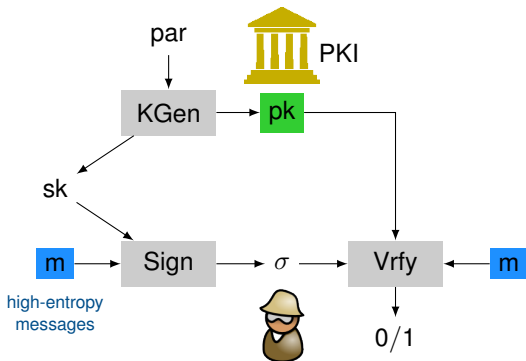
CONF

ANON and CONF are **independent** privacy notions:

- ▶ existence of non-private signature schemes (e.g., FDH-RSA)
- ▶ **black-box separation** of ANON and CONF



# “State-of-the-Art” Privacy Snapshot



Can we achieve ANON and CONF **at the same time?**

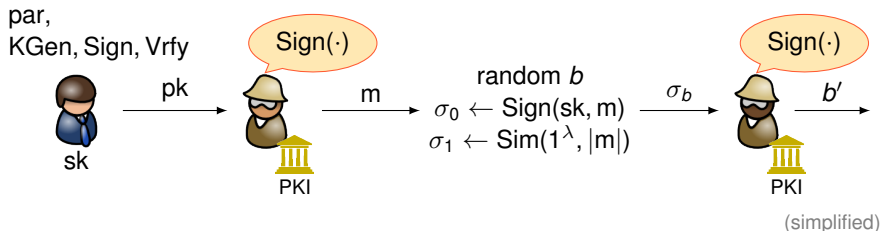


## Intuition

$\text{Sign}(sk, m) \approx \text{Sim}(1^\lambda, |m|)$  — Sim implicitly knows  $(\text{par}, \text{KGen}, \text{Sign}, \text{Vrfy})$

## Indistinguishable Signatures (IND)

There exists a simulator Sim s.t. for all adversaries

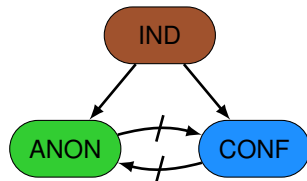
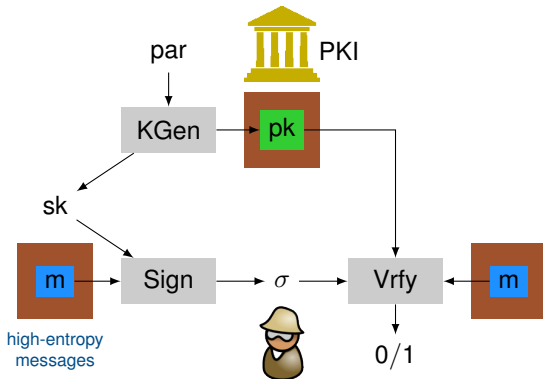


# Examples of IND Signatures

IND signature schemes exist in **different crypto settings**, e.g.,

- ▶ **Probabilistic FDH-RSA with padding**  $\sigma = (\mathbf{H}_N(\mathbf{m}, \mathbf{r})^d + \mathbf{kN}, \mathbf{r})$   
where  $\mathbf{H}_N: \{0, 1\}^* \rightarrow \mathbb{Z}_N$ ,  $\mathbf{k} \in_{\mathbb{R}} [0, \lfloor \mathbf{Z}_\lambda / N \rfloor - 1]$ ,  $\mathbf{Z}_\lambda \in \mathbb{N}$  of  $2\lambda$  bits.
- ▶ **Schnorr scheme** (shared  $\mathbb{G} = \langle g \rangle$ )  $\sigma = (\mathbf{c} = \mathbf{H}(g^r, \mathbf{m}), \mathbf{s} = \mathbf{sk} \cdot \mathbf{c} + \mathbf{r} \bmod \mathbf{q})$   
where  $\mathbf{H}: \{0, 1\}^* \rightarrow \mathbb{Z}_q$  and  $\mathbf{sk} \in_{\mathbb{R}} \mathbb{Z}_q$  is the secret key.
- ▶ **Boneh-Boyen scheme** (shared  $e: \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ )  $\sigma = (g_1^{1/(x+m+yr)}, \mathbf{r})$   
for uniform  $m \in_{\mathbb{R}} \mathbb{Z}_q$  — can be dropped with “hash-then-sign” in ROM.

# “So-Far” Privacy Snapshot



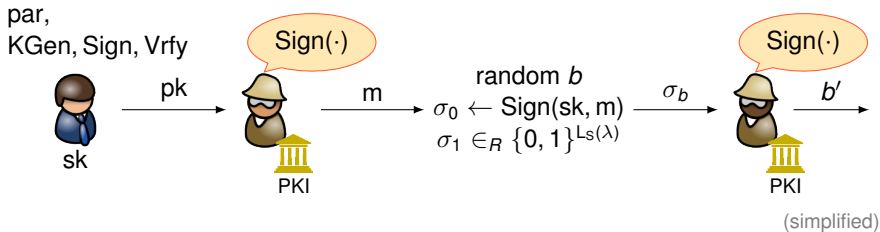
- ▶ **IND signatures may still leak  $par$ !**  
possible to distinguish between security parameters, groups
- ▶ **IND signatures may still leak specification of ( $KGen$ ,  $Sign$ ,  $Vrfy$ )!**  
possible to distinguish between the (instantiations of) schemes

## Intuition

$\text{Sign}(\text{sk}, m) \approx$  random string from  $\{0, 1\}^{L_S(\lambda)}$

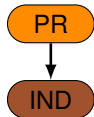
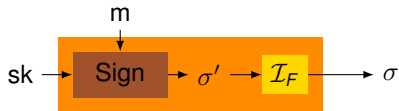
where  $L_S(\lambda)$  is the length of signatures output by a scheme  $S$  on security parameter  $\lambda$

## Pseudorandom Signatures (PR)



Note: Multiple PR signatures can always be extended to some common length  $L$

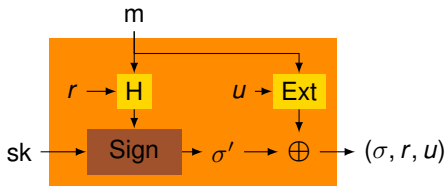
- ▶ converts IND signatures into PR signatures — in the standard model
- ▶ uses **admissible encoding**  $F: \{0, 1\}^{L_S(\lambda)} \rightarrow R$  (Brier et al. @ CRYPTO 2010)
  - ▶  $F$  is **efficient** and **invertible** by  $\mathcal{I}_F$ , which maps to **uniform distribution** in  $\{0, 1\}^{L_S(\lambda)}$
  - ▶ admissible encodings exist for **elliptic curves**,  $\mathbb{Z}_N$ ,  $QR(p)$  and are **aggregatable**



- ▶ suitable for IND schemes with regular Sim (uniform output)
- ▶ aggregation of encodings helps if  $\sigma$  contains elements from various sets
- ▶ **very efficient**

- ▶ works for arbitrary (incl. non-private) signatures — in the standard model
- ▶ bases on construction of ANON signature scheme by Fischlin @ PKC 2007
- ▶ implies that Fischlin's scheme achieves  $PR \Rightarrow IND \Rightarrow CONF$

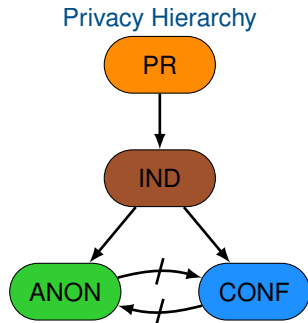
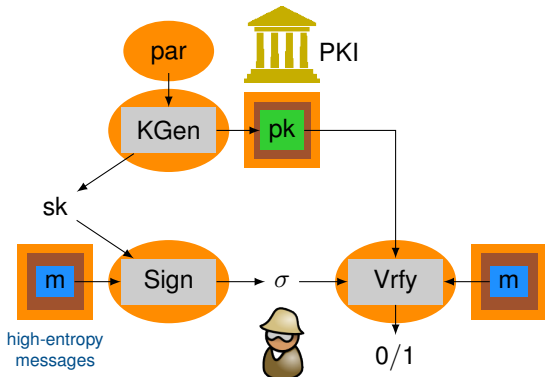
Idea: PR compiler extracts randomness from  $m$  to encrypt the signature



uses associated randomness  
extractor and hash function

$$r, u, H(m, r), Ext(m, u) \approx r, u, H(m, r), v$$

# Final Privacy Snapshot



many of our results also hold in case of full key exposure



## Privacy **is** possible for digital signatures!

- ▶ complete **privacy hierarchy** for signatures ( $PR \Rightarrow IND \Rightarrow \{ANON, CONF\}$ )
- ▶ **constructions** for IND-variants of FDH-RSA, Schnorr, Boneh-Boyen
- ▶ **two generic compilers** (IND-to-PR, direct PR) in the standard model
  
- ▶ **pseudorandom (PR) signatures** hide all information about the signing process — including parameters, instantiations, schemes

All results in our full paper @ <http://eprint.iacr.org/2011/673>, including

- ▶ details on **full key exposure**
- ▶ **impossibility results** for
  - ▶ information recovering signatures (generalization of message recovery)
  - ▶ deterministic signatures



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